Girraween High School

Year 12 Mathematics HSC Trial Examination

2017

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hour
- Calculators and ruler may be used
- All necessary working out must be shown

Total Marks - 100

- Attempt all questions
- Marks may be deducted for careless or badly arranged work
- Start each question on a new sheet of paper

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Question 1 (1 mark)

What is 0.01029 rounded to 3 significant figures?

A. 0.01 B. 0.010 C. 0.0103 D. 0.01029

Question 2 (1 mark)

What are the solutions to the equation $x(x-1) = x$?

A. $x = 0$ B. $x = 1$ C. $x = 0$ and $x = 2$ D. $x = 1$ and $x = 2$

Question 3 (1 mark)

What is the correct factorisation of $8x^3 - 1$?

A. $(2x-1)(4x^2+4x+1)$ B. $(2x+1)(4x^2-4x+1)$ C. $(2x+1)(4x^2-2x+1)$
D. $(2x-1)(4x^2+2x+1)$

Question 4 (1 mark)

What is the angle of inclination of the line $\sqrt{3}x + y = 1$ with respect to the positive x axis?

A. 30° B. 60° C. 120° D. 150°

Question 5 (1 mark)

Which of the following is the domain of the function $f(x) = \dfrac{1}{\sqrt{x+1}}$

A. $x > -1$ B. $x \geq -1$ C. $x < -1$ D. $x \leq -1$

Question 6 (1 mark)

What is the period of the function $f(x) = 2\sin\left(\dfrac{2\pi}{3}x\right)$?

A. 3 B. 3π C. $\dfrac{1}{3}$ D. $\dfrac{\pi}{3}$

Question 7 (1 mark)

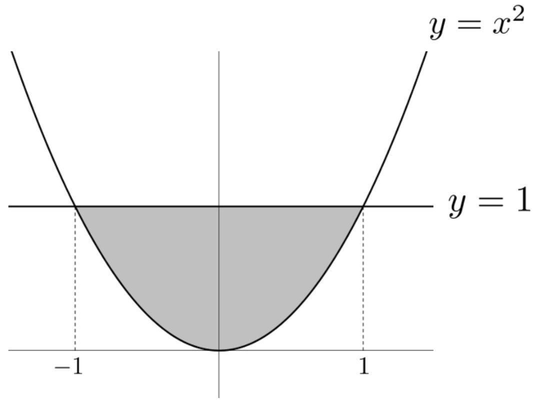
The value of the limit $\lim\limits_{x\to 10} \dfrac{x^2-100}{x-10}$ is

A. undefined B. 0 C. 8 D. 20

Question 8 on the next page

Question 8 (1 mark)

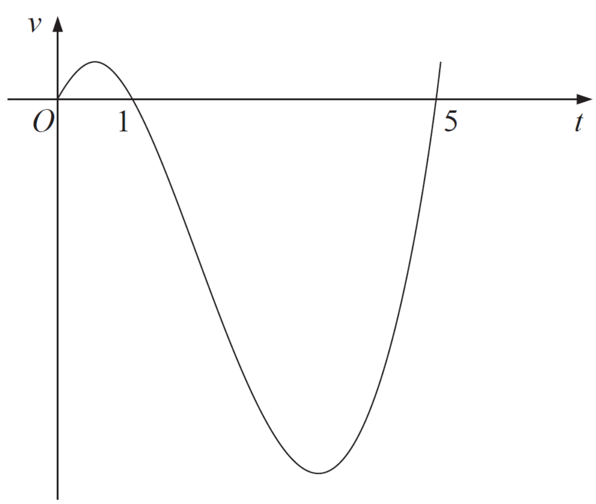
Which of the following integrals does NOT compute the area of the shaded region?



A. $\int_{-1}^{1} x^2 \, dx$ B. $\int_{-1}^{1} 1 - x^2 \, dx$ C. $2 - \int_{-1}^{1} x^2 \, dx$ D. $2 \int_{0}^{1} 1 - x^2 \, dx$

Question 9 (1 mark)

The velocity time graph of a particle travelling in a straight line is given below. Which is the best approximate time at which the particle returns to its initial position?



A. $t = 1$ B. $t = 2$ C. $t = 3$ D. $t = 5$

Question 10 (1 mark)

The position x of a particle travelling in a straight line is given by $x = -2t^2 + t + 1$, which of the following is false in the time frame $0 \leq t \leq 5$?

A. The maximum speed of the particle $>$ the maximum velocity of the particle

B. The minimum speed of the particle $<$ the minimum velocity of the particle

C. The maximum speed of the particle $>$ the minimum velocity of the particle

D. The minimum speed of the particle $<$ the maximum velocity of the particle

Question 11 on the next page

Section II

90 marks

Attempt Questions 11-16

Allow about 2 hours and 45 minutes for this section

Write your answers on the paper provided.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Evaluate $\sqrt{e^2 - 1}$ to two decimal places. [1]

(b) Convert $\dfrac{4\pi}{3}$ radians to degrees. [1]

(c) Find the exact value of $\cos \dfrac{5\pi}{6}$ [1]

(d) Simplify $\dfrac{x}{x(x-1)} + \dfrac{1}{x-1}$ [2]

(e) Find the values of x for which $|x - 2| \geq 5$ [2]

(f) Differentiate with respect to x

i. $y = x \ln x$ [2]

ii. $y = \dfrac{\sin x}{2x}$ [2]

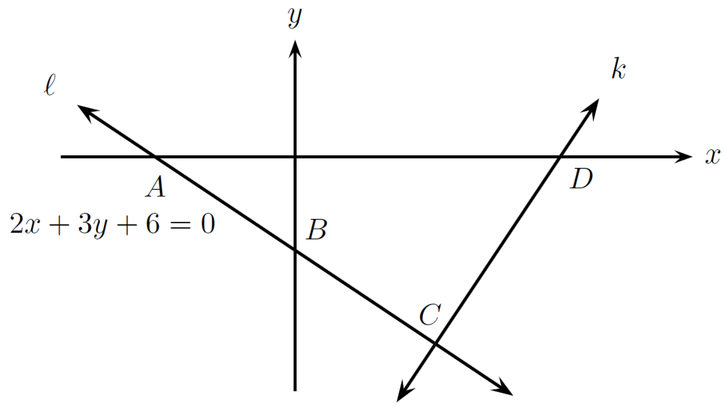
(g) Find $\displaystyle\int \frac{x}{x^2+1}\, dx$ [2]

(h) Find the value of $7 + 14 + 21 + \cdots + 175$. [2]

The exam continues on the next page

Question 12 (15 marks)

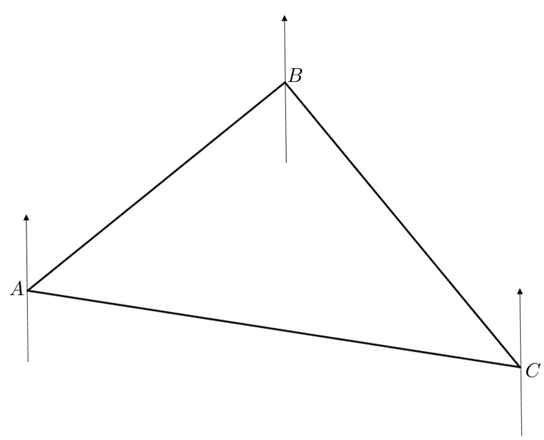
(a) In the diagram below, the line ℓ has equation $2x+3y+6=0$. It cuts the x axis at A and the y axis at B and it intersects the line k at C. Line k is perpendicular to ℓ and cuts the x axis at D.



i. Find the coordinates of A and B. [2]

ii. If B is the midpoint of AC show that the equation of k is given by $3x-2y-17=0$. [3]

(b) In the diagram below, a person walks from point A on a bearing of 030° for 2.4 km to point B. From point B he walks for 3.6 km on a bearing of 145° to arrive at point C.



i. Find the length of AC to one decimal place. [3]

ii. Find the bearing of point C from point A. Give your answer to the nearest minute. [2]

(c) A school sports team has a probability of 0.2 of winning any match.

i. Find the probability the team wins exactly one of its first two matches. [2]

ii. Find the least number of consecutive matches the team must play to be 90% certain that it will win at least one match. [3]

The exam continues on the next page

Question 13 (15 marks)

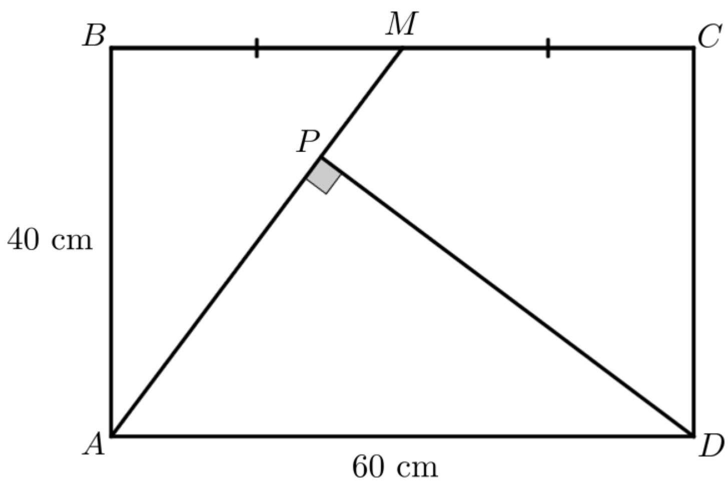
(a) Consider the parabola given by the equation $y = x^2 - 4x + 8$. Find:

i. the coordinates of the vertex. [2]

ii. the coordinates of the focus. [2]

iii. the equation of the directrix. [1]

(b) In the diagram below, $ABCD$ is a rectangle in which $AB = 40$ cm and $AD = 60$ cm. M is the midpoint of BC and DP is perpendicular to AM.



i. Prove that $\triangle ABM \, ||| \, \triangle APD$. [3]

ii. Prove that $PD = 48$ cm. [2]

iii. Find the length of PA and hence show that the area of the quadrilateral $PMCD$ is 936 cm^2. [2]

(c) Consider the series $2 + 4e^{-x} + 8e^{-2x} + \cdots$

i. Show that the series is geometric. [1]

ii. Find the values of x such that this geometric series has an limiting sum. [1]

iii. Find the limiting sum of this geometric series in terms of x. [1]

The exam continues on the next page

Question 14 (15 marks)

(a) Consider the curve $y = x^3 - 3x + 2$.

i. Find the coordinates of the stationary points and determine their nature. [3]

ii. Find any points of inflexion. [2]

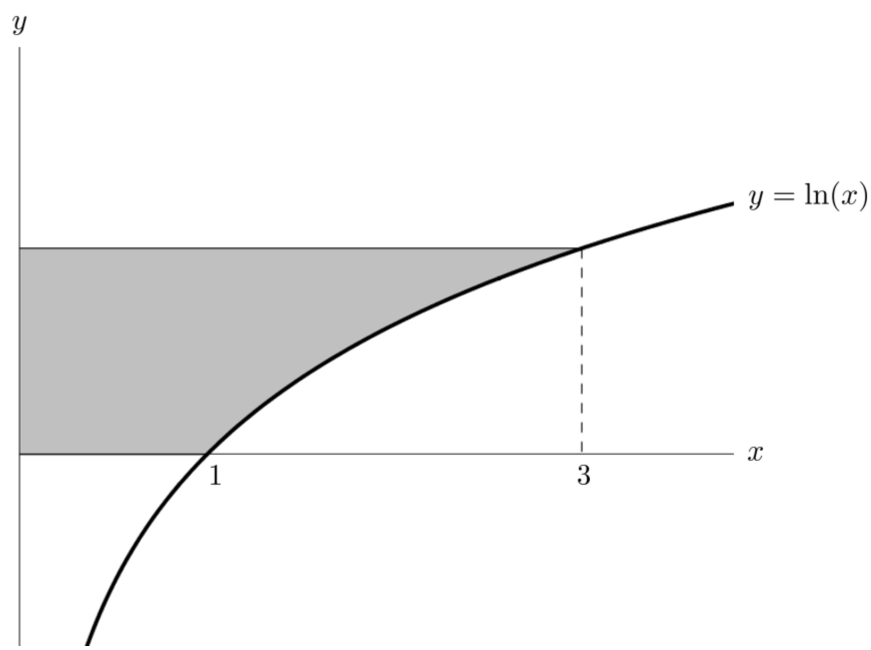
iii. Sketch the curve, showing the stationary points and any points of inflexion. [3]

iv. For what values of x is the curve increasing and concave down? [1]

(b) Use Simpson's Rule with 5 functions values to find an approximation to the value of [3]

$$\int_1^5 \operatorname{cosec} \frac{\pi}{6} x \, dx$$

(c) Find the volume of the solid generated by rotating the shaded area below about the y-axis. [3]



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Question 15 (15 marks)

(a) i. Alan borrows \$10 000 at an interest rate of 12% pa, compounded monthly. Alan wants to pay off the loan with monthly repayments over 5 years. Find the required monthly repayment. [3]

ii. If Alan only wishes to pay \$150 each month over 5 years, how much is he able to borrow under the same interest rate? [2]

(b) A particle travels in a straight line. Its position x metres from the origin is given by

$$x = 6 - 2t + 8\ln(t+2)$$

where time t is measured in seconds.

i. Find the initial velocity and acceleration of the particle. [2]

ii. Find the position of the particle when it is stationary. [2]

iii. State the eventual velocity and acceleration of the particle. [2]

(c) The population N of a certain species at time t is given by

$$N = N_0 e^{-0.03t}$$

where t is in days and N_0 is the initial population of the species.

i. Show that $N = N_0 e^{-0.03t}$ is a solution to the differential equation [1]

$$\frac{dN}{dt} = -0.03N$$

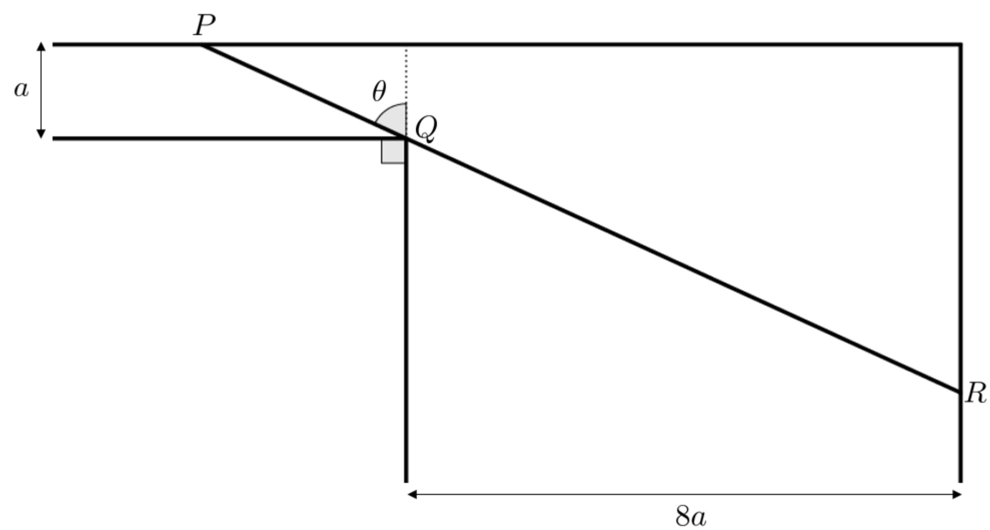
ii. How long will it take for the population to halve? Give you answer to the nearest day. [2]

iii. Find in terms of N_0, the rate of change of the population when the population has halved. [1]

The exam continues on the next page

Question 16 (15 marks)

(a) Two corridors of widths a and $8a$ intersect at a right angle. A straight rope PR touches the corner Q. Let L be the length of PR.



i. Show that $L = \dfrac{a}{\cos\theta} + \dfrac{8a}{\sin\theta}$ [1]

ii. Show that $\dfrac{dL}{d\theta} = \dfrac{a\cos\theta}{\sin^2\theta}(\tan^3\theta - 8)$ [2]

iii. Find the exact value of the minimum value of L. [3]

(b) Suppose α and β are roots of $ax^2 + bx + c = 0$ where $a \neq 0$ and $\alpha \geq \beta$.

i. Show that $\alpha - \beta = \dfrac{\sqrt{\triangle}}{a}$ where $\triangle = b^2 - 4ac$. [1]

ii. Show that $\alpha^2 - \beta^2 = -\dfrac{b\sqrt{\triangle}}{a^2}$. [1]

iii. Show that $\alpha^3 - \beta^3 = \dfrac{\sqrt{\triangle}(b^2 - ac)}{a^3}$. [2]

iv. Show that $\displaystyle\int_\beta^\alpha ax^2 + bx + c\,dx = -\frac{\triangle^{3/2}}{6a^2}$ [3]

(c) Find the values of m such that equation $e^x = mx$ has no solutions. [2]

End of exam

2017 2U TRIALS

C C D C A

A D A B B

Q1

$= 0.0103 \therefore$ (C)

Q2

$x(x-1) = x$

$x^2 - x = x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$\therefore x = 0 \ \& \ x = 2 \ \therefore$ (C)

Q3

$8t^3 - 1 = (2t)^3 - 1$

$= (2t-1)(4t^2 + 2t + 1) \ \therefore$ (D)

Q4

$y = -\sqrt{3}x + 1$

$\therefore \tan\theta = -\sqrt{3} \ \therefore \ \theta = 120° \ \therefore$ (C)

Q5 $x + 1 > 0 \ \therefore x > -1 \ \therefore$ (A)

Q6 $T = \dfrac{2\pi}{\frac{2\pi}{3}} = 3 \ \therefore$ (A)

Q7 $\lim\limits_{x \to 10} \dfrac{(x+10)(x-10)}{x-10}$

$= \lim\limits_{x \to 10} x + 10 = 20 \ \therefore$ (D)

Q8

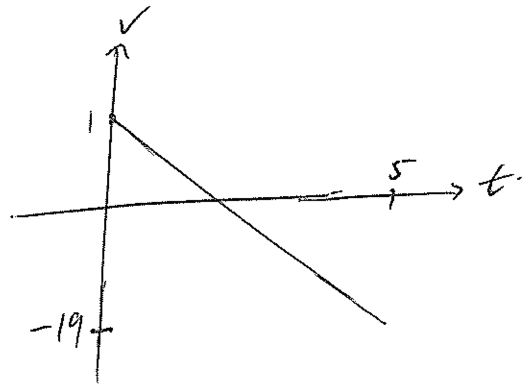
(A)

Q9

$t = 2$ (B)

Q10

$V = -4t + 1$



Max $V = 1$ Max $S = 19$

Min $V = -19$ Min $S = 0$

$\therefore$ (B)

Q11

(a) 2.53 (2dp)

(b) $\dfrac{4 \times 180}{3} = 240°$

(c) $\cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$.

(d) $= \dfrac{x}{x(x-1)} + \dfrac{x}{x(x-1)} = \dfrac{2}{x-1}$

(e) $x - 2 \geq 5$ and $2 - x \geq 5$

$x \geq 7$ and $x \leq -3$

Q11

(f)

(i) $y = x\ln x$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = \frac{1}{x}$

$y' = \ln x + 1$

(ii) $y = \frac{\sin x}{2x}$

$u = \sin x \quad v = 2x$

$u' = \cos x \quad v' = 2$

$y' = \frac{2x\cos x - 2\sin x}{4x^2}$

(g) $\int \frac{x}{x^2+1}\,dx$

$= \frac{1}{2}\int \frac{2x}{x^2+1}\,dx = \frac{1}{2}\ln(x^2+1) + C.$

(h) $7 + 14 + \ldots + 175$

$175 = 25 \times 7$

$\therefore a = 7 \quad n = 25$

$S_{25} = \frac{25}{2}(7 + 175) = 2275$

Q12

(a)(i) $2x + 3y + 6 = 0$

$y = 0, \; x = -3 \; \therefore A = (-3, 0)$

$x = 0, \; y = -2 \; \therefore B = (0, -2).$

(a)

(ii) Let $C = (x, y)$

$\therefore \frac{-3+x}{2} = 0 \quad \& \quad \frac{0+y}{2} = -2$

$\therefore x = 3. \quad \therefore y = -4.$

$\therefore C = (3, -4).$

$3y = -2x - 6$

$y = -\frac{2}{3}x - 2$

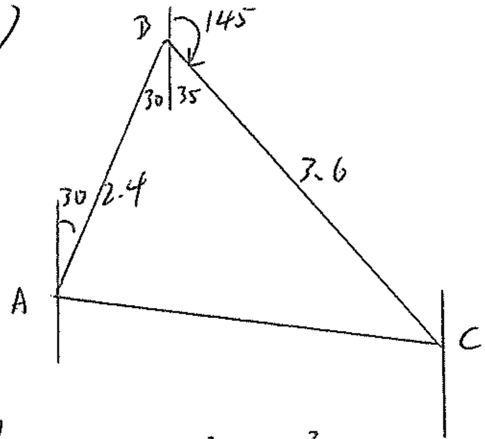
$\therefore m$ of k is $\frac{3}{2}$

$\therefore y + 4 = \frac{3}{2}(x - 3)$

$2y + 8 = 3x - 9.$

$\therefore 3x - 2y - 17 = 0.$

(b)



(i) $AC^2 = 2.4^2 + 3.6^2 - 2 \times 2.4 \times 3.6 \times \cos 65$

$AC = 3.8$ km (1dp)

(ii) $\frac{\sin(\angle BAC)}{3.6} = \frac{\sin 65}{AC}$

$\angle BAC = \sin^{-1}\left(\frac{3.6\sin 65}{AC}\right) = 74°56'$

$\therefore$ Bearing $= 104°56'$

Q12

(c) (i) $P(WL) + P(LW)$

$= 0.2 \times 0.8 + 0.8 \times 0.2$

$= 0.32$

(ii) Let n be the number of matches needed.

$\therefore P(W \text{ at least } 1) = 1 - P(\text{all lose})$

$= 1 - 0.8^n$

We need $1 - 0.8^n = 0.9$

$\therefore 0.8^n = 0.1$

$n = \log_{0.8}(0.1)$

$n = \dfrac{\ln 0.1}{\ln 0.8} = 10.31\ldots$

$\therefore$ need at least 11 games.

Q13

(a)

(i) $y - 8 = x^2 - 4x$

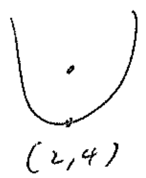
$y - 4 = x^2 - 4x + 4$

$y - 4 = (x-2)^2$

$(x-2)^2 = 4(\frac{1}{4})(y-4)$

$\therefore V = (2, 4)$

(ii)



$\therefore F = (2, 4\frac{1}{4})$

(iii) $y = 3\frac{3}{4}$.

(b)

(i) $\angle ABM = \angle APD = 90$

($\angle$ in a rectangle)

$\angle BMA = \angle PAD$

(alternate $\angle$'s, $BC \parallel AD$)

$\therefore \triangle ABM \parallel\!\parallel \triangle APD$ (Equiangular)

(ii) $BM = 30 \therefore AM = 50$

$\dfrac{PD}{AB} = \dfrac{AD}{AM}$ (ratio of matching sides of $\parallel\!\parallel$ $\triangle$'s)

$\therefore \dfrac{PD}{40} = \dfrac{60}{50}$

$\therefore PD = 48$ cm

(iii) $PA^2 = 60^2 - 48^2$

$\therefore PA = 36$

Area of $PMCD = 40 \times 60 -$ Area of 2 $\triangle$'s

$= 40 \times 60 - \frac{1}{2}(40 \times 30 + 48 \times 36)$

$= 936$

Q13

(c)

(i) $\frac{T_2}{T_1} = \frac{4e^{-x}}{2} = 2e^{-x}$

$\frac{T_3}{T_2} = \frac{8e^{-2x}}{4e^{-x}} = 2e^{-x}$

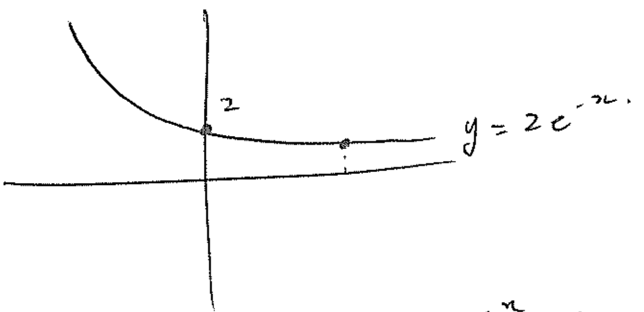
$\therefore r = 2e^{-x}$ $\therefore$ is a GP.

(ii) Need $-1 < r < 1$

$\therefore -1 < 2e^{-x} < 1$

But since $e^{-x} > 0$ for all x

$\therefore -1 < 2e^{-x}$ is true for all x



we check where $2e^{-x} = 1$

$\therefore e^{-x} = \frac{1}{2}$.

$-x = \ln\frac{1}{2}$

$\therefore x = \ln 2$.

$\therefore 2e^{-x} < 1$ when $x > \ln 2$.

$\therefore$ for $x > \ln 2$ limiting sum exists.

(iii)

$S_\infty = \frac{a}{1-r}$

$S_\infty = \frac{2}{1-2e^{-x}}$

$S_\infty = \frac{2e^x}{e^x - 2}$.

Q14

(a)

(i) $y = x^3 - 3x + 2$

$y' = 3x^2 - 3 = 3(x-1)(x+1)$

$y'' = 6x$

$y' = 0$ when $x = \pm 1$

$y(1) = 0 \quad y(-1) = 4$

$\therefore$ Stationary pts are $(1,0)$ & $(-1,4)$

$y''(1) = 6 > 0 \therefore (1,0)$ is min

$y''(-1) = -6 < 0 \therefore (-1,4)$ is max.

(ii) $y'' = 0$ when $x = 0$

$y(0) = 2 \quad \therefore (0,2)$ is possible point of inflexion.

x	-1	0	1
y''	-6	0	6

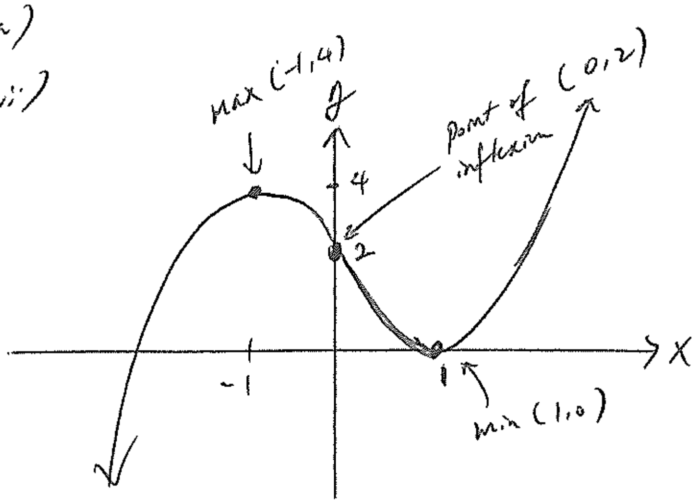
$\therefore$ concavity change

$\therefore (0,2)$ is a point of inflexion.

Q14

(a)

(iii)



(iv) $x < -1$

(b) $h = \frac{5-1}{4} = 1.$

$I \approx \frac{1}{3}\left[f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)\right]$

$\approx \frac{1}{3}\left[2 + 4\left(\frac{2}{\sqrt{3}}\right) + 2(1) + 4\left(\frac{2}{\sqrt{3}}\right) + 2\right]$

≈ 5.08 (2dp).

(c)

$y = \ln x$

$\therefore x = e^y$

$x^2 = e^{2y}$

$V = \pi\int_0^{\ln 3} e^{2y}\,dy$

$V = \frac{\pi}{2}\left[e^{2y}\right]_0^{\ln 3}$

$V = \frac{\pi}{2}\left[e^{2\ln 3} - 1\right]$

$V = \frac{\pi}{2}\left[9 - 1\right] = 4\pi.$

Q15

(a)

(i) Let A_n denote amount owe by end of n^{th} month.

$r = \frac{12}{12} \div 100 = 0.01$

$A_1 = 10000(1.01) - M$

$A_2 = 10000(1.01)^2 - M(1.01) - M$

$A_3 = 10000(1.01)^3 - M(1.01)^2 - M(1.01) - M$

$A_3 = 10000(1.01)^3 - M(1 + 1.01 + 1.01^2)$

$\therefore A_n = 10000(1.01)^n - M(1 + 1.01 + 1.01^2 + \ldots + 1.01^{n-1}).$

$\therefore A_{60} = 10000(1.01)^{60} - M(1 + 1.01 + 1.01^2 + \ldots + 1.01^{59})$

$A_{60} = 0$

$\therefore 10000(1.01)^{60} - M\left[\frac{1.01^{60} - 1}{0.01}\right] = 0$

$\therefore M = \frac{10000(1.01)^{60} \times 0.01}{1.01^{60} - 1}$

$M = \$222.44$

Q15

(a)

(ii) $P(1.01)^{60} - 150\left[\frac{1.01^{60}-1}{0.01}\right] = 0.$

$\therefore P = \frac{150(1.01^{60}-1)}{0.01 \times (1.01)^{60}}$

$P = \$6743.26$

(b)

(i) $x = 6 - 2t + 8\ln(t+2)$

$v = -2 + \frac{8}{t+2} = -2 + 8(t+2)^{-1}$

$a = -8(t+2)^{-2} = -\frac{8}{(t+2)^2}$

$v(0) = 2 \text{ ms}^{-1}$

$a(0) = -2 \text{ ms}^{-2}$

(ii) $v = 0$ when $-2 + \frac{8}{t+2} = 0.$

$\therefore t+2 = 4 \quad \therefore t = 2.$

$x(2) = 6 - 4 + 8\ln 4$

$x = 2 + 8\ln 4 \text{ m}$

(iv) as $t \to \infty$ $v \to -2 \text{ ms}^{-1}$

$a \to 0 \text{ ms}^{-2}$

Q15

(c)

(i) $N = N_0 e^{-0.03t}.$

$\frac{dN}{dt} = -0.03 \times N_0 e^{-0.03t}.$

$\therefore \frac{dN}{dt} = -0.03N$

(ii) $\frac{N_0}{2} = N_0 e^{-0.03t}.$

$\therefore e^{-0.03t} = \frac{1}{2}.$

$-0.03t = \ln\frac{1}{2}$

$0.03t = \ln 2$

$t = \frac{\ln 2}{0.03} = 23.104\ldots$

$\therefore$ 24 days needed.

(iii) $\frac{dN}{dt} = -0.03N$

So when $N = \frac{N_0}{2}$ $\frac{dN}{dt} = -0.03 \times \frac{N_0}{2}$

$\frac{dN}{dt}. = \frac{3}{200}N_0$

(iv)

Q16

(a)

(i) $\cos\theta = \frac{a}{PQ}$ $\quad \sin\theta = \frac{8a}{QR}$

$\therefore PQ = \frac{a}{\cos\theta}$ $\quad \therefore QR = \frac{8a}{\sin\theta}$

$L = PQ + QR = \frac{a}{\cos\theta} + \frac{8a}{\sin\theta}$.

(ii) $L = a(\cos\theta)^{-1} + 8a(\sin\theta)^{-1}$

$\frac{dL}{d\theta} = -a(\cos\theta)^{-2} \times -\sin\theta$
$\quad - 8a(\sin\theta)^{-2} \times \cos\theta$

$\frac{dL}{d\theta} = \frac{a\sin\theta}{\cos^2\theta} - \frac{8a\cos\theta}{\sin^2\theta}$.

$\frac{dL}{d\theta} = \frac{a\cos\theta}{\sin^2\theta}\left(\frac{\sin^3\theta}{\cos^3\theta} - 8\right)$

$\frac{dL}{d\theta} = \frac{a\cos\theta}{\sin^2\theta}(\tan^3\theta - 8)$.

(iii) $\frac{dL}{d\theta} = 0$ when $\tan^3\theta - 8 = 0$

$\therefore \tan\theta = 2$.

note that $\cos\theta \neq 0$ for $0 < \theta < \frac{\pi}{2}$

$\therefore \theta = \tan^{-1}2$ gives the stationary point.

Now as $\tan\theta$ is an increasing function

if $\theta > \tan^{-1}2$ then $\tan\theta > 2$

$\therefore \tan^3\theta > 8$

$\therefore \frac{dL}{d\theta} > 0$

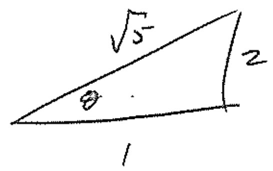
if $\theta < \tan^{-1}2$ then $\tan\theta < 2$

$\therefore \tan^3\theta < 8$

$\therefore \frac{dL}{d\theta} < 0$

$\therefore \theta = \tan^{-1}2$ gives min L.

Now if $\tan\theta = 2$



$\therefore \sin\theta = \frac{2}{\sqrt{5}}$ & $\cos\theta = \frac{1}{\sqrt{5}}$

$L = \frac{a}{\cos\theta} + \frac{8a}{\sin\theta}$

$\therefore L = \sqrt{5}a + 8a\left(\frac{\sqrt{5}}{2}\right)$

$L = \sqrt{5}a + 4a\sqrt{5}$

$L = 5a\sqrt{5}$.

(b)

(i) $\alpha = \frac{-b+\sqrt{\Delta}}{2a}$ $\quad \beta = \frac{-b-\sqrt{\Delta}}{2a}$

$\therefore \alpha - \beta = \frac{-b+\sqrt{\Delta}+b+\sqrt{\Delta}}{2a}$

$= \frac{\sqrt{\Delta}}{a}$.

(ii) $\alpha^2 - \beta^2 = (\alpha-\beta)(\alpha+\beta)$

$= \frac{\sqrt{\Delta}}{a} \times \frac{-b}{a} = \frac{-b\sqrt{\Delta}}{a^2}$.

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(b)

(iii)

$$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$$

$$= (\alpha-\beta)(\alpha^2+2\alpha\beta+\beta^2-\alpha\beta)$$

$$= (\alpha-\beta)\left[(\alpha+\beta)^2-\alpha\beta\right]$$

$$= \frac{\sqrt{\Delta}}{a}\left[\frac{b^2}{a^2}-\frac{c}{a}\right]$$

$$= \frac{\sqrt{\Delta}}{a}\left(\frac{b^2-ac}{a^2}\right) = \frac{\sqrt{\Delta}(b^2-ac)}{a^3}$$

(iv)

$$\int_\beta^\alpha ax^2+bx+c\,dx$$

$$= \left[\frac{ax^3}{3}+\frac{bx^2}{2}+cx\right]_\beta^\alpha$$

$$= \frac{a\alpha^3}{3}+\frac{b\alpha^2}{2}+c\alpha-\frac{a\beta^3}{3}-\frac{b\beta^2}{2}-c\beta$$

$$= \frac{a}{3}(\alpha^3-\beta^3)+\frac{b}{2}(\alpha^2-\beta^2)+c(\alpha-\beta)$$

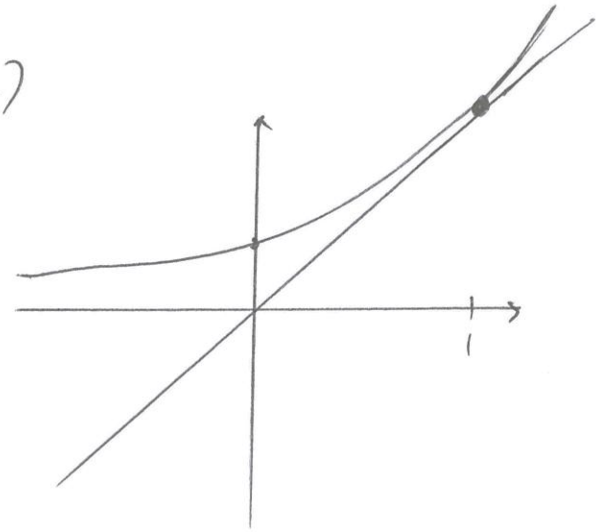
$$= \frac{a}{3}\times\frac{\sqrt{\Delta}(b^2-ac)}{a^3}-\frac{b}{2}\times\frac{b\sqrt{\Delta}}{a^2}+c\left(\frac{\sqrt{\Delta}}{a}\right)$$

$$= \frac{\sqrt{\Delta}}{3a^2}(b^2-ac)-\frac{b^2\sqrt{\Delta}}{2a^2}+\frac{c\sqrt{\Delta}}{a}.$$

$$= \frac{2\sqrt{\Delta}b^2-2ac\sqrt{\Delta}-3b^2\sqrt{\Delta}+6ac\sqrt{\Delta}}{6a^2}$$

$$= \frac{4ac\sqrt{\Delta}-b^2\sqrt{\Delta}}{6a^2} = \frac{\sqrt{\Delta}(4ac-b^2)}{6a^2} = \frac{-\Delta^{3/2}}{6a^2}$$

(c)



We first find the value of m such that $y=mx$ is tangent to $y=e^x$

$\therefore\ m=e^x \quad \cdots ①$

$mx=e^x \quad \cdots ②$

$\frac{②}{①}$ gives $x=1$

$\therefore\ m=e.$

$\therefore$ no solutions i.e. no intersections when

$$0\le m<e$$